ROUND I: Definitions

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. The successor of an integer is the next larger integer. The square of a certain integer is 721 less than the square of its successor. Find the integer.

2. If a and b are two real numbers, then $a ¤ b$ is defined to be $ab + 7a + 7b + 42$. Find a number e such that $a ¤ e = a$ for all values of a.

3. If $x \, \Delta \, y = \begin{cases} y \text{ if } x + y > 0 \\ x \text{ if } x + y \leq 0 \end{cases}$

   and $x \lozenge y = \begin{cases} 2y \text{ if } x - y > 0 \\ 2x \text{ if } x - y \leq 0 \end{cases}$

   Evaluate $[\, 2 \, \Delta \, ((-2 \, \Delta \, 4) \lozenge (-3))] \lozenge (-5)$.

ANSWERS

(1 pt.) 1.__________________________

(2 pts) 2.__________________________

(3 pts) 3.__________________________

Auburn, Burncoat, Hudson, Quaboag

April 1, 1998 WOCOMAL VARSITY MEET

ROUND II: Algebra I - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Terry starts walking home from school at 4 miles per hour. Ten minutes later, Chris leaves school, running to catch up with Terry. What speed does Chris have to average in order to catch up with Terry in 15 minutes?

2. Solve $\sqrt{x+3} = 9 - x$

3. There are three numbers greater than one such that:
   The difference of the reciprocals of the two smaller is $\frac{3}{40}$
   The sum of the reciprocals of the smallest and largest is $\frac{29}{120}$
   The difference of the reciprocals of the two larger is $\frac{1}{12}$
   Find the three numbers.

ANSWERS

(1 pt.) 1.____________________mi/hr

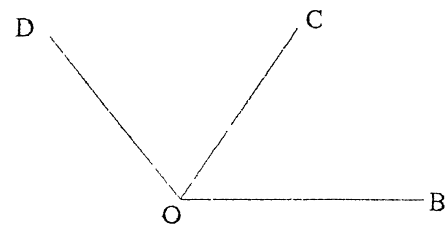
(2 pts) 2.____________________________

(3 pts) 3.____________________________

Bancroft, Quaboag, South

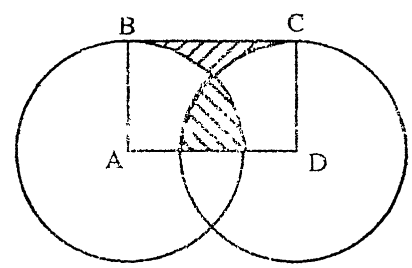
ROUND III: Geometry - open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM



1. If $m\angle DOB = 4x - 1$
   $m\angle DOC = 2x + 6$
   and $m\angle COB = 59°$,
   Find $m\angle DOB$ in degrees.

2. A new sports field is to be sodded using 2ft by 2ft squares of sod. If the length of the field is 70 yards longer than the width and its area is 6000 sq. yards, how many squares of sod will be needed?



3. In circles A and D, AB = DC = 1 and $\overline{BC}$ is a tangent segment. If the shaded regions have equal areas, find length BC. (If π is involved, keep it as π. Don't approximate.)

ANSWERS
(1 pt.) 1.______________________________

(2 pts) 2.______________________________

(3 pts) 3.______________________________

Algonquin, Southbridge, Tahanto

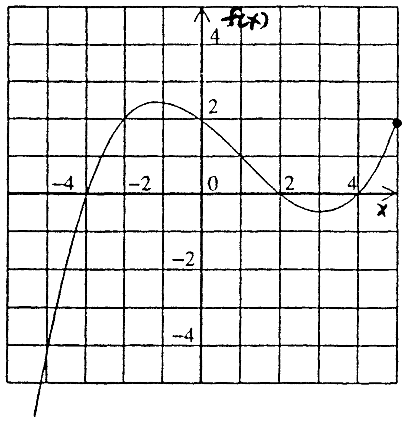
April 1, 1998 WOCOMAL VARSITY MEET

ROUND IV: Functions

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If $f(x) = \frac{2}{3}x + 6$, find $f(x+3) - f(x)$

2. For $f(x)$ shown by the graph at the right, solve $f(2x) = 2$, for x.



3. Find $f(6)$ given

$$f(x) = \begin{cases} f(f(x-2)) + 1 \text{ when } x > 1 \\ 2 \text{ when } x = 1 \\ 1 \text{ when } x = 0 \end{cases}$$

ANSWERS

(1 pt.) 1.____________________________

(2 pts) 2. $x =$ __________________________

(3 pts) 3.____________________________

Auburn, Bancroft, Mass Academy

April 1, 1998 WOCOMAL VARSITY MEET

ROUND V: Trigonometry - open

EACH ANSWER MUST BE IN THE FORM SPECIFIED IN THE PROBLEM

1. Find the smallest positive value of x in radians for which $\sin 2x = 2\sin x$.

2. On March 21 on a certain island on the equator, the sun rises due east at 6am, is directly overhead at noon, and sets due west at 6pm (locally). At 2pm a tree casts a 15 foot long shadow. To the nearest foot, how long will the shadow be at 5pm? Assume flat ground.

3. All three sides of ΔABC have integer lengths. The shortest side is 4 units long. What is the longest possible side that ΔABC can have and still be an acute, non-isosceles triangle?

ANSWERS

(1 pt.) 1.________________________

(2 pts) 2.____________________ft___

(3 pts) 3.________________________

Mass Academy, St. John's, QSC

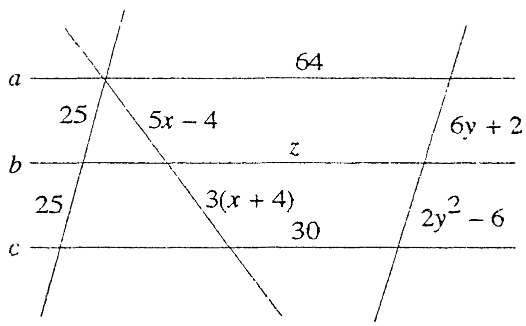
TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THE SEPARATE TEAM ANSWER SHEET

2 pts. each

1. Let $S(N)$ be the number of ways to represent a positive integer N as a sum of two or more consecutive positive integers in increasing order. For example: $4+5 = 9$ and $2+3+4 = 9$, so $S(9) = 2$. Find $S(63)$.

2. 5 picas have the same value as 13 mortites, 5 mortites have the same value as 13 fedoras, and 5 fedoras have the same value as 13 minskies. How many minskies will you need to have the same value as 1000 picas?

3. 

If lines a, b, and c are parallel, find the sum $x + y + z$.

4. Suppose $f(x) = \frac{1}{\sqrt{x}}$, $x > 0$, and $g(x) = \cos x$, $-\pi \leq x \leq \pi$

Find the domain of $f\left(g\left(x - \frac{\pi}{2}\right)\right)$

5. Evaluate $\cot\left[\arcsin\left(\tan\left[\arccos\frac{-12}{13}\right]\right)\right]$ in simplified radical form. A fraction is acceptable, but no decimal approximation.

6. If the geometric mean of p and q is 3, and the sum of their squares is 8, evaluate $(p + q)^2$

7. The product of a set of distinct positive integers is 48. What is the smallest possible sum of such a set of integers?

8. The planes defined by the equations $4x + y + 2z = 1$ and $x + 4y = 0$ both include the point $(k, k^2, 2k^3 + 1)$. Find k.

9. Five integers a, b, c, d, and e are such that $a < b < c < d < e$. Also if any two of them are added, the sum is either 43, 46, 49, 50, 51, 53, 54, 56, 57, or 61. Find the value of c.

Assabet Valley, Auburn, Burncoat, Hudson, Quaboag, St. John's, Shrewsbury, Westborough

April 1, 1998 WOCOMAL VARSITY MEET ANSWERS

ROUND I defs	1. 1 pt	360
	2. 2 pts	-6
	3. 3 pts	-10
ROUND II alg 1	1. 1 pt	$\frac{1}{9}$ mi/min $6\frac{2}{3}$ or $\frac{20}{3}$ or $6.\overline{6}$ $\frac{mi}{hr}$
	2. 2 pts	6
	3. 3 pts	5, 8, 24 any order
ROUND III geom	1. 1 pt	$131°$
	2. 2 pts	13,500
	3. 3 pts	$\frac{\pi}{2}$
ROUND IV logs, exp, radicals	1. 1 pt	32
	2. 2 pts	$\frac{1}{2}$ or $.5$
	3. 3 pts	$a^2, \frac{1}{a}$ (need both)
ROUND V trig	1. 1 pt	π
	2. 2 pts	97 ft
	3. 3 pts	8

TEAM ROUND 2 pts each

1. 5
2. 17,576
3. 59
4. $0 < x < \pi$ or $(0, \pi)$
5. $\frac{-\sqrt{119}}{5}$
6. 26
7. 12
8. $-\frac{1}{4}$ or $-.25$
9. 26